Migration modelling in the New Economic Geography

C. Camacho

Discussion Paper 2010-4

Institut de Recherches Économiques et Sociales
de l'Université catholique de Louvain

IRES

UCL

Migration modelling in the New Economic Geography*

Carmen Camacho†

February 15, 2010

Abstract

The benchmark of this paper is the Fujita and Thisse (2002) core-periphery model, which adds a R&D sector with skilled labor to create new varieties for the modern sector. The number of R&D firms increases not only with the number of existing patents and knowledge spillovers but also with the number of skilled workers who can migrate and choose the region offering the better lifetime salary. The main objective of the present work is to analyse the long-term consequences of the choice of the migration law in Fujita and Thisse (2002) and in other comparable models. After describing throughoutly our benchmark, we introduce a different migration law *à* la Krugman (1991). Although the change in the migration law implies that individuals do not foresee price changes and hence their choice is somehow less optimal, the steady state outcome does not vary qualitatively: the unique steady state is a symmetric distribution of labor across regions. Later we change the benchmark model to avoid the so called monotonic convergence hypothesis, about which we discuss at large in the paper. When we model the economy using Romer (1990) two sector model applied to two regions allowing for skilled migration, then there exists a solution path that converges to a steady state which exhibits a distribution of skilled workers amongst regions which is no longer symmetric. In effect, the new steady state depends on technology, fixed costs, knowledge spill-overs and transportation costs.

Keywords: Economic geography, Spatial Dynamics, Migrations, Growth.

*We acknowledge the financial support of the Belgian French speaking Community (convention ARC 09/14-019) on "Geographical Mobility of Factors". I thank Fabio Mariani for valuable comments.

†Carmen Camacho is the corresponding author. E-mail address: carmen.camacho@uclouvain.be

1 Introduction

"Increasing returns in production activities are needed if we want to explain economic agglomerations without appealing to the attributes of physical geography. In particular, the trade-off between increasing returns in production and transportation costs is central to the understanding of the geography of economic activities",

(p.7, Fujita and Thisse, 2002).

Neoclassical growth models explain the intertemporal evolution of economies leaving aside the location issue. Almost all the research carried in geographic economics has used models with productive externalities. On the technical side of the modelling, this choice multiplies the number of equilibria, introducing the problem of equilibrium selection into the discussion. What are the forces leading one economy towards an equilibrium rather than another? What is more relevant in the long-term, initial endowments or self-fulfilling expectations? If it was the initial distribution of wealth, then nothing can prevent a region poorly endowed from a poverty trap. Among these equilibria, we usually observe total agglomeration in one region or configurations in which marginal prices are equated across space.

In economies with Marshalian externalities, productive factors move towards those activities yielding initially the largest marginal returns. In this sense, history matters, determines the economy dynamics and its long-run outcome. Expectations do not play a major role since in this environment, decisions are made according to current returns. Nevertheless, the role of expectations is undeniable in other settings where future prospects affect current decisions. Furthermore, what matters is not only the expectations about the future value of earnings but also how expectations themselves are formed.

It is sensitive then to ask whether population movements would reinforce or modify the economic predictions of the Economic Geography models. Migration is a complex phenomenon involving numerous interacting aspects. Individuals leave their cities, their culture, language, family, habits and experience. There are also costs associated to moving, searching for a job and a house. Some would argue that there exist assimilation costs for the host region when origin and destination are culturally afar. Nevertheless inter-regional movements do not necessarily imply a cultural difference or a language change. Probably for this reason, movements across regions outsize international ones.

Given the importance of migrations in an increasingly interdependent world where capital is freely mobile, a number of authors have proposed economic models with labor movements. Many of these models are developed within a partial equilibrium framework, where the dynamic consequences of migrations on the regional economy at origin and destination cannot be studied. These models are frequently static, losing consequently the power to analyze the short and medium term consequences of migrations. Few models encompass migrations and their effects on the whole economy in both the short and the long term.

We believe that the modelling of inter-regional migrations needs a more comprehensive appraisal within a general equilibrium dynamic model. We describe three models where migrations are introduced in such framework. In section 3 we reproduce in detail the model in Fujita and Thisse (2002). The authors propose a general equilibrium model to study the links between agglomeration and growth in a two regions framework with monopolistic competition. Only skilled workers are allowed to migrate. Economic growth is driven by R&D which is itself developed by skilled workers' knowledge. There is an externality in the production of patents or ideas, which is translated into researchers' salaries. This in turn leads to a medium-term agglomeration of the R&D activity in one region.

The counterpart of increased salaries are costs: migrants pay a cost which is a congestion cost following Musa (1978). This cost is a loss in utility which depends on the rate of migration inasmuch as a migrant imposes a negative externality on the others. The more people change region, the larger it is. This modeling of costs smoothes the dynamics of migrations and it avoids bang-bang solutions. They are related to regional price indexes reflecting the loss in utility due to differences in life-costs. Consequently, the assumption of perfect foresighted agents implies that future regional wages and prices must be known before taking the migration decision. The migration law depends on future regional wages and prices. A classical dynamic analysis is not possible in this framework. To cope with this restriction, the authors introduce the *monotonic convergence hypothesis*. From all possible expectations for perfect-foresighted agents, they pick the one that yields monotonic convergence from the initial condition to the long-run solution. More specifically, they fix the initial and final distributions of skilled workers across regions.

We provide detailed comments on some aspects of this model at the end of the section. We propose in section 4 a simplification of the model. We replace the original migration law by that of Krugman (1991). The main difference with the previous migration law is that agents do not take into account how life-costs are modified with population changes through goods demand, individuals are less-perfect foresighted. With this simpler migration law we are able to analyze the dynamic and long-run properties of the model using standard techniques. The unique steady state is the symmetric distribution of skilled individuals across regions. However, this long-run outcome is unstable and the solution path reaches a corner solution, that is agglomeration of skilled individuals in one of the regions.

Next, in section 5 we present a different framework to analyze migrations based on Romer (1990). Here there is a unique type of individual who is able to work in the two economic sectors: manufacturing and R&D. There is an externality in the production of new patents, namely productivity is boosted by past ideas. As long as salaries are affected by the number of varieties in the regional economies, workers choose the region with a longer R&D history. In contrast to Fujita and Thisse, the unique steady state is not symmetric but a distribution of workers which depends on research fixed costs, on regional technologies and on transportation costs. It is the regional configuration where marginal prices and returns are equalized. More importantly, the resulting dynamic system has a stable manifold. This means that there exists a set of initial conditions along a stable manifold such that the solution path reaches its steady state. We present our conclusions in the final section of the paper.

2 Related literature

As mentioned before, this paper stems from the work of Fujita and Thisse (2002). This is a complete and complex model in which skilled individuals are allowed to migrate. The symmetric distribution of individuals is the unique steady state of the model, although the only two long-run stable configurations are the corner solutions, that is total agglomeration of skilled individuals in one region.

Krugman (1991) and its sequel by Fukao and Benabou (1993) model inter-sectorial labor movements rather than inter-regional, but their migration law can be adapted to the latter. They assume that agents are somehow myopic, since when taking their migration decision they do not account for the consequences of this decision on regional prices. The unique steady state is unstable. The solution path diverges from its steady state value towards one of the corner solutions forming spirals or monotonically depending on the parameter set and the initial conditions.

Mossay (2001) studies the role of expectation formation in convergence speed. He explores the same two country model with perfect foresighted and myopic agents. If the countries differ largely in their initial endowments, then rational adjustments can lead the economies to converge faster to their long-run configuration than with myopic adjustments.

In the three models studied in this paper, we only allow skilled workers to migrate. As stated by empirical research the propensity to migrate increases with education. Our assumption is then a simplification of reality that focus on the most sizeable movement. According to Greenwood (1997) migration rises with education. He shows using data for US flows between 1980 and 1985, that migration propensities are highest for the group of individuals between 25 and 29 years old. For all age classes, migration propensity increases with education. For the group from 25 to 29 years old with five or more years of college, it is 4.6 times as high as for individuals with 0 to 8 years of elementary school (and 2 times as high for the 45 to 64 years old group). For Canada similar results are shown by Ledent (1990). Antolin and Bover (1997) study inter-regional migration in Spain using data from the Spanish Migration Survey, from 1987 to 1991. Their results are in line with Greenwood's. After controlling for marital status, the number of children, being head of family, regional unemployment rates, etc. they find that "higher education increases the probability of migration and people with primary education are the less prone to migrate..." (p. 221, Antolin and Bover, 1997). Gianetti (2001) studies the links between migration and education in Italy. She also finds a positive relationship between education and migration. Besides, the percentage of skilled individuals that have completed college or high-school in a region represents a pull factor for skilled individuals in other regions.

A piece of research close to ours is provided by Gianetti (2003). She introduces migrations in a general equilibrium growth model. Individuals' skills are complementary *à la* Kremer (1993), which creates a self-selection of migrants. If the skill premium is increasing with the average human capital, then skilled workers have an incentive to move to the rich regions. Low skilled workers would leave rich regions to the poor regions to minimize their living costs. Individuals compare the utility provided by their next period salary in both regions, taking into account that they must fulfill a minimum level for consumption to cover basic needs. This minimal consumption level is then reduced to housing prices, that could also have been interpreted as a migration fixed cost. She characterizes the equilibrium, which can be asymmetric depending on the skill distribution: all skilled individuals move to the North and all unskilled to the South.

3 A benchmark model for inter-regional migration

In Fujita and Thisse (2002) model (FT hereafter) there are two regions, region 1 and region 2, and three economic sectors: the traditional good ($\mathbb{T}$) sector, the modern ($\mathbb{M}$) sector and the R&D sector.

Let $L_{T,j}$ be the number of workers in the $\mathbb{T}$ sector, L_j the number of unskilled workers in the $\mathbb{M}$ sector and λ_j the number of skilled workers in region j. Besides, we assume for simplicity reasons that population is constant and that $L_{T,1}(t)+L_{T,2}(t)+L_1(t)+L_2(t) = L$ and $\lambda_1(t) + \lambda_2(t) = 1$ for any $t \geq 0$. Only skilled individuals are allowed to migrate, therefore $L_{T,1}(t) + L_1(t) = L/2$ and $L_{T,2}(t) + L_2(t) = L/2$, $\forall t$.

The population is formed by infinite lived dynasties. Agents have the following instantaneous utility function

$$u(T,Q) = \frac{T^{1-\mu}Q^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}}, \tag{1}$$

where T is the amount of traditional good consumed, Q is an index of final or manufactured goods defined as $Q = \left(\int_0^M q_i^{\rho} di\right)^{\frac{1}{\rho}}$, M is the number of final good varieties and q_i is the quantity consumed of final good i. If ϵ stands for the individual's earnings, then her budget constraint is

$$\epsilon = T + \int_0^M p_i q_i di, \tag{2}$$

The agents' problem consists in maximizing their instantaneous utility (1) given their budget constraint (2). Notice that infinite lived dynasties maximize their present consumption not taking into account their future revenue. The first order conditions with respect to T and q_i for all $i \in [0, M]$ yield the optimal quantities of the traditional and the modern goods:

$$T = \frac{1-\mu}{\mu}\int_0^M p_i q_i di = (1-\mu)\epsilon, \tag{3}$$

and

$$q_i(t) = \mu\epsilon p_i(t)^{-\sigma} P^{\sigma-1}, \tag{4}$$

where σ is defined as $\sigma \equiv \frac{1}{1-\rho}$ and $P(t) = \left(\int_0^M p_i(t)^{-(\sigma-1)} di\right)^{-\frac{1}{\sigma-1}}$ is the price index of the $\mathbb{M}$ varieties. Since $0 < \rho < 1$, we obtain that $1 < \sigma < \infty$. Substituting (3) and (4) into (1), we can compute the indirect utility of an agent living in region r $v_r(\epsilon_r(t)) = \epsilon_r(t)P_r(t)^{-\mu}$, where P_r is the price index in region r. The lifetime utility of an individual is defined by

$$U(0) = V(0) - \Sigma_h e^{-\gamma t_h} C_m(t_h), \tag{5}$$

with γ the subjective discount rate common to all individuals, t_h the moment when the hth migration takes place and C_m moving costs associated to migration. Notice that migration costs depend on the time the migration takes place. The lifetime utility gross of migration costs, $V(0)$, is defined as

$$V(0) = \int_0^\infty e^{-\gamma t} \ln(v(t))dt.$$

There is a global and perfectly competitive market, where bonds are traded bearing an interest rate $\nu(t)$ at time t. This interest rate is common to the two regions. We denote by $\bar{v}(t,0) = \frac{1}{t}\int_0^t v(\tau)d\tau$ the average interest rate in period $[0,t]$. If $w_r(t)$ stands for the wage received in region r at time t, then the present value of the wage income is given by

$$W(0) = \int_0^\infty e^{-\bar{\nu}(t,0)t} w_r(t)dt.$$

The consumer intertemporal budget constraint is then

$$\int_0^\infty \epsilon(t)e^{-\bar{\nu}(t,0)t}dt = a_H + W(0), \tag{6}$$

being a_H the value of the consumer's initial assets. Maximizing the individual's lifetime utility in (5) subject to her intertemporal budget constraint (6) we obtain that $\frac{\dot{\epsilon}(t)}{\epsilon(t)} = \nu(t) - \gamma$, which implies that the dynamics of regional expenditure E are $\frac{\dot{E}(t)}{E(t)} = \nu(t) - \gamma$. There exists a traditional good produced with constant returns to scale, T. Only unskilled labor is used in its production and their salary equals their marginal productivity, that is $w_T(t) = 1$ for all $t \geq 0$. As a consequence, the total production of the traditional good is $L_T(t)$.

Let us assume that once a $\mathbb{M}$ firm buys from the R&D sector the patent to produce a new final good, it produces it with constant returns to scale. In region j there are M_j final good firms, each one producing a different good. We shall drop for the moment being regional indexes for the sake of simplicity.

The firm producing good i employs l_i unskilled workers. Hence $L_j(t) = \int_0^{M_j(t)} l_i(t)di$ defines total labor in the final good sector in region j, for $j = 1,2$. Production of firm j is $Y_j(t) = l_j(t)$ so that $w_j(t) = 1$. Thus, unskilled workers receive a unit salary in both regions and sectors.

Demand for final good i produced in region 1 is:

$$Q_i(t) = \mu E_1(t)p_i(t)^{-\sigma}P_1^{\sigma-1} + \mu E_2(t)p_i(t)^{-\sigma}\Upsilon^{1-\sigma}P_2^{\sigma-1}, \tag{7}$$

and $\Upsilon > 1$ stands for transportation costs of the iceberg type. Transportation costs account for the amount of good that "melts" in the way.

At equilibrium $Q_i(t) = Y_i(t)$ for all i and t. Then final good firm i maximizes its profit:

$$\max_{Q_i} (p_i(t) - 1)Q_i(t),$$

which yields $p_i(t) = \frac{1}{\rho}$, $\forall i \in [0, M]$. Setting $\phi = \Upsilon^{1-\sigma}$, we obtain the following identity for regional prices

$$P_r(t) = \frac{1}{\rho} (M_r + \phi M_s)^{\frac{-1}{\sigma-1}}, \tag{8}$$

for $r, s \in \{1, 2\}$, $r \neq s$. Firms' profits are given by $\pi_r = \frac{Q_r}{\sigma-1}$. Substituting (8) into (7):

$$Q_r(t) = \mu\rho \left(\frac{E_r}{M_r + \phi M_s} + \frac{\phi E_s}{M_s + \phi M_r} \right), \tag{9}$$

with $s, r \in \{1, 2\}$ with $r \neq s$.

R&D productivity increases with the existing capital of past ideas and methods, and this capital is seen as a public good[1]. New ideas in region j, n_j are produced according to

$$n_j = K_j \lambda_j, \tag{10}$$

where K_j is knowledge capital in region j and λ_j skilled workers in region j. Besides, there are some knowledge spillovers between researchers in both regions, measured by the parameter η. Denoting by h individual's human capital, K_j is defined as

$$K_j = \left(\int_0^{\lambda_r} h(z)^\beta dz + \eta \int_0^{\lambda_s} h(z)^\beta dz \right)^{\frac{1}{\beta}}.$$

If we further assume that individual human capital equals the total number of patents $h = M$, then

$$K_r = Mk\left(\lambda + \eta(1 - \lambda)\right), \tag{11}$$

where $k(\cdot)$ is a strictly convex and increasing function, which verifies that $k(0) = 0$ and $k(1) = 1$. Patents growth is then $\dot{M} = n_1 + n_2$. Replacing (10) and (11) into the later expression for $\dot{M}$

$$\dot{M} = M\left(\lambda k\left(\lambda + \eta(1 - \lambda)\right) + (1 - \lambda)k\left(1 - \lambda + \eta\lambda\right)\right) = Mg(\lambda). \tag{12}$$

[1] Time indexes are not shown for the ease of exposition.

Consequently, $g(\lambda) = \lambda k\left(\lambda + \eta(1-\lambda)\right) + (1-\lambda)k\left(1-\lambda+\eta\lambda\right)$ is the patent production growth rate. Let us denote by $k_1(\lambda) = k\left(\lambda + \eta(1-\lambda)\right)$ and $k_2(\lambda) = k\left(1-\lambda+\eta\lambda\right)$.

The cost of a new invention in region r is $w_r/Mk_r(\lambda)$, where $Mk_r(\lambda)$ measures the average productivity of a researcher in region r. Profits in the R&D sector, Π_j, equal benefits obtained by the final good firm that has developed the patent:

$$\Pi(t) = \int_t^{\infty} \pi^*(\tau)e^{-\gamma(\tau-t)}d\tau.$$

At equilibrium costs in the R&D sector equal its benefits, equating these quantities we obtain that salaries in the sector are $w_r = \Pi_j Mk_r(\lambda)$. If final good firms are footloose, then profits are equal for all firms. Since at equilibrium firm's profits are $\pi_j = Q_j/(1-\sigma)$, $Q_i = Q_j$ for any i and j.

3.1 Migration behavior

The counterpart of enjoying larger lifetime earnings are migration costs which depend on the size of the migratory flow as in Mussa (1978):

$$C_m(t) = \frac{|\dot{\lambda}(t)|}{\delta}. \tag{13}$$

In order to construct the migration law and due to the perfect foresight hypothesis, the *monotonic convergence hypothesis* (*mc-hypothesis* hereafter) is introduced:

Definition 1 [Monotonic Convergence Hypothesis]
Let $\tilde{\lambda} \in [0,1]$ and $\lambda_0 \in [0,1]$ such that $\tilde{\lambda} \neq \lambda_0$. If $\{\lambda(t)\}_{t=0}^{\infty}$ is an equilibrium path satisfying the initial condition $\lambda(0) = \lambda_0$, this path satisfies the monotonic convergence hypothesis *under $\tilde{\lambda}$ when $0 \le T \le \infty$ exists such that*

$$\begin{array}{lll} \text{when } \lambda_0 < \tilde{\lambda} & \dot{\lambda}(t) > 0 \text{ for } t \in (0,T), \\ & \lambda(t) = \tilde{\lambda} \text{ for } t \ge T, \\ \text{when } \lambda_0 > \tilde{\lambda} & \dot{\lambda}(t) < 0 \text{ for } t \in (0,T), \\ & \lambda(t) = \tilde{\lambda} \text{ for } t \ge T. \end{array}$$

Therefore, the mc-hypothesis restricts expectations in trajectories that converge towards a long-run solution, which is equivalent to restricting equilibrium paths to monotonic paths. In other words, individuals migrate only once. Notice that through the mc-hypothesis not only the authors focus on perfect foresighted expectations, but they pick a very specific type of expectation function. Hence, there is no room for stochastic behaviors, coordination failures, etc.

Lifetime earnings of an individual migrating from region 2 to region 1 at time t are defined as

$$W(0;t) = \int_0^t w_2(s)e^{-\gamma s}ds + \int_t^\infty w_1(s)e^{-\gamma s}ds,$$

and using (5) and (13), her lifetime utility is

$$U(0;t) = V(0;t) - e^{-\gamma t}\frac{|\dot{\lambda}(t)|}{\delta},$$

that is migratio cost is a loss in utility which depends on the migration rate. In effect, each migrant imposes a negative externality on the others. At equilibrium migrants are indifferent about their migration time, that is $U(0;t) = U(0;T)$. After some substitutions, the migration law obtains:

$$\dot{\lambda}(t) = \frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{a_H + W(0;t)}{a_H + W(0;T)}\right) + \delta\mu e^{\gamma t}\int_t^T e^{-\gamma s}\ln\left(\frac{P_2(s)}{P_1(s)}\right)ds. \tag{14}$$

3.2 Equilibrium

Total production of traditional good is L_T, then the traditional good market clears when

$$L_T = (1-\mu)E(t). \tag{15}$$

Since $L_1 + L_2$ is the final good sector production:

$$L_1 + L_2 = \mu\rho E(t). \tag{16}$$

Putting together (15) and (16) we obtain that total expenditure is constant:

$$E = \frac{L}{1-\mu(1-\rho)}. \tag{17}$$

Then if λ is fixed, regional expenditure is

$$E_r = \frac{L}{2} + \lambda_r\left(a_H + \frac{w_r(\lambda_r)}{\gamma}\right) = \frac{L}{2} + \lambda_r a_H\left(\gamma + k_r(\lambda_r)\right) \tag{18}$$

for $r = 1,2$. Besides, (18) is also used to compute regional expenditure when λ is not fixed.

Then, using that at equilibrium $Q_rM_r = L_r$ and $Q_1 = Q_2 = Q$, we have that $Q = \frac{\mu\rho L}{M(1-\mu(1-\rho))}$.

3.3 Main results

There are three different equilibria depending on the relationship between $\frac{E_1}{E_2}$ and ϕ. If $\phi < \frac{E_1}{E_2} < \frac{1}{\phi}$, $M_r = M\frac{E_s - \phi E_r}{(1-\phi)E^*}$. If $\frac{E_1}{E_2} \geq \frac{1}{\phi}$, then $M_1 = M$ and $M_2 = 0$. Otherwise if $\phi \geq \frac{E_1}{E_2}$, $M_1 = 0$ and $M_2 = M$. In the three cases, $\pi^* = \frac{\mu E}{\sigma M}$.

When FT study the dynamics of the model, they provide the following definition of *stability*:

Definition 2 [Stability]
The ss-growth path under $\tilde{\lambda}$ is said to be stable *if there exists a neighborhood Λ of $\tilde{\lambda}$ such that, for any $\lambda_0 \in \Lambda$ with $\lambda_0 \neq \tilde{\lambda}$, an equilibrium path exists that satisfies the monotonic convergence hypothesis under $\tilde{\lambda}$. The ss-growth path under $\tilde{\lambda}$ is said to be* unstable *when there is no such neighborhood of $\tilde{\lambda}$.*

Under the mc-hypothesis there are three values of $\bar{\lambda}$ under which the economy is at a steady state: $\bar{\lambda} = 0$, $\bar{\lambda} = 1/2$ and $\bar{\lambda} = 1$. Agglomeration of skilled workers in a region yields a stable steady state, whereas the symmetric distribution does not.

As the following proposition puts forward, even when the R&D sector agglomerates, the final good sector can have a different distribution.

Proposition 1 [Agglomeration results]
When firms are completely mobile, then the stable spatial configuration exhibits

i) a dominant agglomeration involving the innovation sector entirely and a large fraction of the modern sector in the same region when

$$\Upsilon^{\sigma-1} > \frac{\sigma+\mu}{\sigma-\mu};$$

ii) a global agglomeration involving the innovation and the modern sectors in the same region when

$$\Upsilon^{\sigma-1} \leq \frac{\sigma+\mu}{\sigma-\mu},$$

Proof: See chapter 11 in Fujita and Thisse (2002).

3.4 Some comments

Krugman's (1991) article is part of a literature devoted to the study of the roles of history and self-fulfilling expectations in the case of multiple equilibria. As it was put forward by Matsuyama (1991), in the case of highly non-linear models, there are multiple steady states due to externalities. Though local analysis can be conducted, a global analysis is required to rule out other perfect foresight solution paths. In Krugman (1991) history or self-fulfilling expectations determine the long-run solution depending on the parameter set.

Another example of a model with productive externalities is the previously mentioned by Matsuyama (1991). We can observe that results are not as simple as in Krugman (1991). The debate surrounds FT's model. The externalities present in the R&D sector cause the multiplicity of steady state solutions and the existence of a set of self-fulfilling expectations leading to either long-run outcome. Furthermore, this assumption also allowed the model to reach monotonically a long-run solution in finite time given that population is constant.

We believe that closer attention has to be paid to the dynamics of FT model in two aspects. First, we focus on the dynamic equations provided for regional expenditure and total patent production. Individual expenditure depends on the individual's trajectory as the equation shows:

$$\epsilon_j = \gamma \left(a_H + W_j(0)\right)$$

where $W_j(0)$ is individual's j total life-time earnings. Hence, individual expenditure is conditioned to migration movements. As a consequence regional expenditure at time t, which is the addition of all expenditure quantities realized by individuals living in the region at that moment, is a function of the particular migration trajectory of each individual and of the moment they arrived to the region. This is why regional expenditure would be better modelled as

$$\begin{aligned} E_1(t) = \frac{L}{2} + \int_0^t \gamma\dot{\lambda}_1(s)\left(a_H + W(0,s)\right) I_{\dot{\lambda}_1(s)>0} ds + \\ + \int_t^T \gamma(-\dot{\lambda}_1(s))\left(a_H + W(0,s)\right) I_{\dot{\lambda}_1(s)<0} ds + \lambda_1(0)\gamma\left(a_H + W(0)\right), \quad (19) \end{aligned}$$

and $E_2(t) = E - E_1(t)$. $I_{\dot{\lambda}_r(s)>0}$ is the indicator function for $r = 1, 2$ whose value is one if $\dot{\lambda}_r(s) > 0$ and zero otherwise. $W(0)$ stands for lifetime earnings of individuals who do not move from region 1 and $W(T)$ for lifetime earnings of those who do not leave region 2. The first integral term in E_1 represents the amount spent by individuals who have arrived to region 1 before time t. The second integral term is the amount spent by that those live in region 1 at time t but who migrate sometime after t. The last term in (19) is the amount spent at time t by individuals who do not leave region 1.

Regional expenditure appears indirectly in the migration law, and given its definition in (19), the agglomeration results in section 3.3 many not hold anylonger. Then, even at steady state, when $\lambda = \bar{\lambda}$, we are not able to compute E_r. Consequently, it is almost impossible to check under which conditions the ratio E_1/E_2 is larger or smaller than ϕ or its inverse, since this ratio varies with time and it depends on the future of variable λ as well as on its past. Besides, as times evolves the economy could shift from an equilibrium to another.

We know from equation (12) that the dynamics of M are described by

$$\dot{M}(t) = g(\lambda(t))M(t),$$

which implies that

$$M(t) = M(0)e^{\int_0^t g(\lambda(s))ds}. \tag{20}$$

before λ reaches a constant value. Nevertheless it is true that once the steady state or any other long-run solution is attained, the dynamics of M are described by $\dot{M}(t) = g(\bar{\lambda}(t-T))M(T)$ being T the exact time at which a long-run solution is achieved. Then, we use that at equilibrium R&D firm's profits are equal to compute salaries:

$$\Pi(t)M(t) = \int_t^\infty \frac{\mu E}{\sigma M(\tau)} M(t) e^{-\gamma(\tau - t)} d\tau =$$

$$= \frac{\mu E}{\sigma} \int_t^\infty e^{-\int_t^\tau (\gamma + g(\lambda(s)))ds} d\tau.$$

Finally, we have attempted to solve the dynamics of this problem using standard techniques (see appendix 7 for technical details). If $\phi < \frac{E_1}{E_2} < \frac{1}{\phi}$, we can build a system of four dynamic equations in four variables $\{\lambda, E_1, \Theta, \Lambda\}$. Θ is the second term in (14) and $\Lambda(t)$ represents lifetime earnings of someone migrating to region 1 at time t. If λ attains a constant value, so does E_1, and Θ is constant by construction at T. However, Λ grows forever at a decreasing rate, which hardens the analysis except if $\lambda = 1/2$ was a stable steady state.

If $\frac{E_1}{E_2} \geq \frac{1}{\phi}$, then $\dot{\lambda}(T) = 0$ by construction but $\lambda = 1/2$ is no longer a steady state.

4 Modifying the migration law

In this section, we modify FT's model using the migration law presented by Mussa (1978), retaken afterwards by Krugman (1991).

4.1 Krugman's migration law

Mussa (1978) and Krugman (1991) modelled the movement of individuals between sectors as a flow responding to market incentives. Population changes were described by the incentive to work in a sector rather than in other. This vision differs from FT in many respects. First, agents do not compare utilities but the salaries' value. Second, agents are "less" perfect foresighted than in FT, i.e. when they decide to migrate they do not consider the possibility of future migrations nor the effect of their move on the future of the economy. It is assumed that agents believe that they will only move once. In Krugman (1991) there exists a region for the parameters and the initial conditions such that convergence to the steady state is not monotonic but in spirals. This means that agents actually move more than once.

We shall implement Mussa, Krugman migration law into FT's model. This amounts to model migrations as:

$$\dot{\lambda}(t) = \beta \left(\int_t^\infty w_1(s) e^{-\gamma(s-t)} ds - \int_t^\infty w_2(s) e^{-\gamma(s-t)} ds \right),$$

where $\lambda(0)$ is known and $\lim_{t \to T} \dot{\lambda}(t) = 0$. To simplify the dynamic analysis, we define $q(t)$ as

$$q(t) = \int_t^\infty w_1(s) e^{-\gamma(s-t)} ds - \int_t^\infty w_2(s) e^{-\gamma(s-t)} ds,$$

and $\lim_{t \to \infty} q(t) = 0$. Then

$$\dot{\lambda}(t) = \beta q(t), \tag{21}$$

$$\dot{q}(t) = \gamma q(t) - \Pi(t) M(t) \left[k_1(\lambda(t)) - k_2(\lambda(t)) \right], \tag{22}$$

where q is understood by Krugman as the opportunity cost of investing one unit of labor in region 1 rather than in region 2. We define $\Pi(t)$ as in FT

$$\Pi(t) = \int_t^\infty e^{-\gamma(\tau - t)} \pi^*(\tau) d\tau.$$

At equilibrium, instantaneous profits are $\pi^*(t) = \frac{\mu E(t)}{\sigma M(t)}$. Then, since total expenditure is constant and $M(\tau) = M(t)e^{g(\lambda(\tau - t))}$:

$$\Pi(t)M(t) = \frac{\mu E}{\sigma} \int_t^\infty e^{-\int_t^\tau (\gamma + g(\lambda(s)))ds} d\tau.$$

Then, if a steady state is reached, $\Pi(t)M(t)$ equals $\Pi(t)M(t) = \frac{\mu E}{\sigma} \frac{1}{\gamma + g(\overline{\lambda})}$.

The introduction of Krugman (1991) migration law allows for an easier analysis of the dynamics of migrations.

Proposition 2 [Steady State]
If we denote by $V(t) = \Pi(t)M(t)$, we obtain the following dynamic system:

$$\begin{cases} \dot{\lambda}(t) = \beta q(t), \\ \dot{q}(t) = \gamma q(t) - V(t) \left[k_1(\lambda(t)) - k_2(\lambda(t)) \right], \\ \dot{V}(t) = (\gamma + g(\lambda(t)))V(t) - \frac{\mu E}{\sigma}. \end{cases} \tag{23}$$

There exists an interior steady state to the system described by the set $\{\bar{\lambda} = 1/2, \bar{q} = 0, \bar{V} = \frac{\mu E}{\sigma} \frac{1}{\gamma + g(1/2)}\}$. The system diverges from its unique interior steady state and it reaches a corner solution $\overline{\lambda} = 0$ or $\overline{\lambda} = 1$ in finite time since population is finite.

Proof: See appendix 8.

Linearizing (23) around the steady state, we obtain the associated eigenvalues: $l_1 = \gamma + g(1/2)$ and $l_{2,3} = \frac{\gamma \pm \sqrt{\gamma^2 - 8\beta(1-\eta)\gamma k'(\frac{1+\eta}{2})}}{2}$. Independently of the sign of the radical in $l_{2,3}$, the three eigenvalues have a positive real part. Hence convergence to one of the corner solutions is monotone if $\gamma > 8\beta(1-\eta)k'(\frac{1+\eta}{2})$ and history determines the final outcome. The existence of an overlap depends on γ, η, β and $k(\cdot)$, or in other words, on the time discount rate, the strength of knowledge spillovers and on externalities in R&D production. If individuals heavily discount the future, then as in Krugman (1991), there is no overlap. Similarly, if externalities are not large enough (measured by η, β and $k(\cdot)$), then individuals' decisions do not depend on the others. In this aspect, our results follow the lines in Krugman (1991) and Fukao and Benabou (1993) (for a discussion on the subject see next subsection).

4.2 Comparing results

A noticeable feature of this model is that results are in line with FT's. More precisely, under Krugman's migration law agents do not take into consideration the regional price changes. If these two models predict similar properties for the solution trajectory, it means that the price change caused by migration plays no significant role.

Next, we compare the model results in this section with the original model in Krugman (1991). In Krugman (1991) there exists a steady state distribution of workers between the two sectors which depends on salary formation in the sector whose production is affected by externalities. This steady state is unstable and the solution trajectory reaches in finite time a corner solution. Whether one or the other solution is reached depends first on the parameter set. The relevant parameters for stability are r, the interest rate, γ the speed of adjustment and β the strength of externalities.

If $r^2 > 4\gamma\beta$, then the system steadily diverges from the steady state to reach a long-run solution which is completely determined by history. Otherwise there exists a couple of complex roots. In the case of complex roots, there exists a range of expectations that are decisive for the long-run rather than history. Krugman remarks that there exists a range of initial values for which this is true and he call this range the *overlap*. If there is no overlap, then history always decides the long-run outcome. The existence and width of the overlap depend on r, γ and β. If r is sufficiently large, then there is no overlap. If the future is heavily discounted, individuals do not care about others' future actions and then it is history that determines the long-run. A small β means that externalities are not large enough, then decisions will not be interdependent. If γ is small, economies adjust slowly and history is always decisive.

We would like to study the distribution of manufacturing firms between the two regions. In this model, the equilibrium is defined by equations (15), (16), (17) and (19) together with $QM_r = L_r$ with $Q_1 = Q_2 = Q = \frac{\mu\rho L}{M(1-\mu(1-\rho))}$. Since $E_1 + E_2 = E$, and $M_1 + M_2 = M$ and $Q_r(t) = \mu\rho\left(\frac{E_r}{M_r+\phi M_s} + \frac{\phi E_s}{M_s+\phi M_r}\right)$, then $M_1 = \frac{E_1-\phi E_2}{(1-\phi)E}M$ and $M_2 = \frac{E_2-\phi E_1}{(1-\phi)E}M$. Consequently, our analysis has the same three types of equilibria as FT. However we cannot disentangle the relationship between transportation costs and agglomeration once we have opted for (19) to define regional expenditure.

Therefore, we have improved the dynamic analysis by modifying the migration law though we cannot obtain clear results about manufacturing agglomeration. We shall try to improve simultaneously both aspects in next section, modifying completely the theoretical framework.

5 Changing functional forms

We apply the framework developed by Romer (1990) to an economy made of two regions. There are two sectors in each region: the final good sector and the R&D sector. R&D produces patents for intermediate goods and develops these goods. Then, all intermediate goods are employed in the production of the final good. All individuals earn the same salary in the region, independently of the sector they work in. It is assumed that population is constant. Even if all individuals in the same region earn the same salary, we decide to restrict migration only to individuals in the R&D sector to keep comparability with FT.

There is a final good firm in each region that produces using labor and all intermediate goods[2]:

$$Y_i = A_i L_i^{1-\alpha} \sum_{j=0}^{N} (x_{ij})^{\alpha},$$

where the subscript makes reference to the region. A_i is the technology coefficient in region i, α satisfies that $0 < \alpha < 1$, L_i is labor in the final good sector, $x_{i,j}$ is the quantity of intermediate good j used in the production of the final good in region i. Finally, N is the total number of intermediate goods. Then, the final good firm in region i maximizes its profits:

$$\max_{L_i} A_i L_i^{1-\alpha} \sum_{j=0}^{N} (x_{ij})^{\alpha} - w_i L_i - \sum_{j=0}^{N} p_j x_{ij}.$$

where w_i is the salary in region i and p_j the price of intermediate good j. Taking the first order conditions we obtain the optimal salary and the optimal quantity of each intermediate good:

$$w_i = A_i (1-\alpha) L_i^{-\alpha} \sum_{j=0}^{N} (x_{ij})^{\alpha},$$

$$x_{ij} = \left(\frac{A_i \alpha}{p_j} \right)^{\frac{1}{1-\alpha}} L_i.$$

Intermediate good j has price p_j and to produce it, there is a cost equal to one. Total demand for intermediate good j produced in region 1 is:

$$X_1 = \left(\frac{A_1 \alpha}{p_j} \right)^{\frac{1}{1-\alpha}} L_1 + \left(\frac{A_2 \alpha}{\Upsilon p_j} \right) L_2 \tag{24}$$

[2]We change FT's specification for the production of the final good. Keeping their specification would imply a unit salary in both regions and therefore there wouldn't exist any incentive to migrate.

where Υ accounts for transportation costs of the iceberg type and $\Upsilon > 1$. Similarly, total demand for any intermediate good produced in region 2

$$X_2 = \left(\frac{A_1\alpha}{\Upsilon p_j}\right)^{\frac{1}{1-\alpha}} L_1 + \left(\frac{A_2\alpha}{p_j}\right)^{\frac{1}{1-\alpha}} L_2, \tag{25}$$

The R&D firm that retains the monopoly power over the patent it invents, maximizes the profit function $(p_j - 1)X_j$ at each period. Taking the first order conditions we obtain the optimal value for the intermediate good price, $p_j = \frac{1}{\alpha}$.

Let us assume that all intermediate goods are available in both regions. Since all intermediate goods have the same price, the final good firm uses the same quantity of the intermediate goods coming from one region. Nevertheless, regions use more important quantities of the goods produced at home since they do not incur in a transportation cost[3].

Consumers spend all their revenue in the final good, which is unique though produced in two different regions. Then the final good market clears when

$$Y_1 + Y_2 = (L_1 + \lambda_1)w_1 + (L_2 + \lambda_2)w_2 = l_1 w_1 + l_2 w_2, \tag{26}$$

where l_i is total population in region i, for $i = 1, 2$.

In Romer (1990) the cost of a new idea is proportional to the number of existing ideas, namely $w_r \frac{\eta_r}{N_r + \theta_s N_s}$ with η_r the fixed cost of R&D in units of goods, θ_r the knowledge spill-over from region r to region s and N_r the number of patents developed in region r. At equilibrium costs equal profits. Hence for a R&D firm in region r:

$$w_r \frac{\eta_r}{N_r + \theta_s N_s} = \int_t^\infty (p_r - 1) X_r e^{-\bar{r}(v,t)(v-t)} dv, \tag{27}$$

p_r is the price of an intermediate good developed in region r and $\bar{r}(v,t)$ is the average interest rate between times v and t defined as $\bar{r}(v,t) = \frac{1}{v-t}\int_t^v r(s)ds$.

Substituting w_r, p_r and X_r into (27):

$$A_r^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(N_r + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_s)\frac{\eta_r}{N_r + \theta_s N_s} =$$

$$= \int_t^\infty \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}} \left(A_r^{\frac{1}{1-\alpha}} L_r + \Upsilon^{\frac{-1}{1-\alpha}} A_s^{\frac{1}{1-\alpha}} L_s\right) e^{-\bar{r}(v,t)(v-t)} dv, \tag{28}$$

[3]Consequently, regional final goods could be considered heterogenous since they are produced following different procedures. Nevertheless, we do not consider this alternative and we assume that final goods are homogenous for the sake of simplicity.

For computational reasons, it is convenient to assume that $\theta_r = \Upsilon^{\frac{-\alpha}{1-\alpha}}$, for $r = 1, 2$. This implies that spillovers are identical from both regions.

Since migration is not allowed for individuals in the $\mathbb{M}$ sector, L_1 and L_2 are constant. Therefore we obtain that

$$A_1^{\frac{1}{1-\alpha}} \frac{\eta_1}{\alpha} = \left(A_1^{\frac{1}{1-\alpha}} L_1 + \Upsilon^{\frac{-1}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} L_2 \right) \int_t^\infty e^{-\bar{r}(v,t)(v-t)} dv, \tag{29}$$

and

$$A_2^{\frac{1}{1-\alpha}} \frac{\eta_2}{\alpha} = \left(A_2^{\frac{1}{1-\alpha}} L_2 + \Upsilon^{\frac{-1}{1-\alpha}} A_1^{\frac{1}{1-\alpha}} L_1 \right) \int_t^\infty e^{-\bar{r}(v,t)(v-t)} dv. \tag{30}$$

The value of R&D firms is identical in both regions and the interest rate r is constant and equal to

$$r = \frac{\alpha}{\eta_1} \frac{A_1^{\frac{1}{1-\alpha}} L_1 + \Upsilon^{\frac{-1}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} L_2}{A_1^{\frac{1}{1-\alpha}}}. \tag{31}$$

Dividing (29) by (30) we obtain the equilibrium relationship between L_1 and L_2:

$$\frac{L_1}{L_2} = \frac{A_2^{\frac{1}{1-\alpha}} \left(\eta_1 A_1^{\frac{1}{1-\alpha}} - \eta_2 \Upsilon^{\frac{-1}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} \right)}{A_1^{\frac{1}{1-\alpha}} \left(\eta_2 A_2^{\frac{1}{1-\alpha}} - \eta_1 \Upsilon^{\frac{-1}{1-\alpha}} A_1^{\frac{1}{1-\alpha}} \right)} \tag{32}$$

As in Romer (1990) dynamics of R&D in region i are described by

$$\dot{N}_i = \frac{N_i + \theta_j N_j}{\eta} \lambda_i,$$

for $i = 1, 2$ with $i \neq j$ and where λ_i is the number of workers in the R&D sector in region i.

Consumers are endowed with a fixed quantity of labor that they supply inelastically. They receive the wage rate w on their fixed supply of labor. At $t = 0$ they own the existing durable-good producing firms, and they earn the rate of return r on these assets. If we denote by a the individual's assets, the problem each infinitely lived agent faces is

$$\max_c \int_0^\infty \left(\frac{c^{1-\sigma} - 1}{1 - \sigma} \right) e^{-\gamma t} dt,$$

subject to their budget constraint:

$$\dot{a}(t) = ra(t) + w(t) - c(t), \tag{33}$$

where $c(t)$ is the quantity of consumption at time t. Solving this optimal control problem yields the optimal growth rate of consumption:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left(r - \gamma\right), \tag{34}$$

and a transversality condition for c, $\lim_{t\to\infty} u'(c(t))e^{-\rho t} = 0$.

The budget constraint (33) expresses the financial restrictions faced at one point in time by a consumer, who maximizes her lifetime utility. In FT, individuals maximize their lifetime utility, net of migration costs, subject to their lifetime budget constraint (6).

5.1 Equilibrium

Total population is constant. L_1, L_2 workers in the final good sector are fixed and $\lambda_1+\lambda_2 = 1$. The final good market clears when $Y_1 + Y_2 = l_1 w_1 + l_2 w_2$. Equilibrium salaries are given by

$$w_i = A_i^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1-\alpha) \left(N_i + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_j\right), \tag{35}$$

for $i = 1, 2$, $i \neq j$. Total individual's savings are equal to investment, and hence

$$\dot{N} \frac{w_1 \eta_1}{N_1 + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_2} = \dot{N} \frac{w_2 \eta_2}{N_2 + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_1}.$$

Total household's assets equal the market value of the firms, i.e.

$$N \frac{w_1 \eta_1}{N_1 + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_2} = N \frac{w_2 \eta_2}{N_2 + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_1}.$$

Regional production of the final good is

$$Y_1 = A_1^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_1 \left(N_1 + N_2 \Upsilon^{\frac{-\alpha}{1-\alpha}}\right), \tag{36}$$

and

$$Y_2 = A_2^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_2 \left(N_2 + N_1 \Upsilon^{\frac{-\alpha}{1-\alpha}}\right). \tag{37}$$

Total demand for each intermediate good are:

$$X_1 = \left(A_1 \alpha^2\right)^{\frac{1}{1-\alpha}} L_1 + \left(\frac{A_2 \alpha^2}{\Upsilon}\right) L_2 \tag{38}$$

and

$$X_2 = \left(\frac{A_1 \alpha^2}{\Upsilon}\right)^{\frac{1}{1-\alpha}} L_1 + \left(A_2 \alpha^2\right)^{\frac{1}{1-\alpha}} L_2. \tag{39}$$

5.2 Migration

Modelling migration as in Krugman (1991) enables us to study the dynamic properties of the model. In this model agents compare the discounted logarithm of salaries as a utility measure[4] instead of comparing the values of salaries. If q stands for the "shadow price of the asset of having a unit of labor" in region 1 rather than in region 2, then

$$\dot{\lambda}_1(t) = \delta \left(\int_t^{\infty} \ln(w_1(s)) e^{-\gamma(s-t)} ds - \int_t^{\infty} \ln(w_2(s)) e^{-\gamma(s-t)} ds \right), \tag{40}$$

or defining $q(t)$ as

$$q(t) = \int_t^{\infty} \ln(w_1(s)) e^{-\gamma(s-t)} ds - \int_t^{\infty} \ln(w_2(s)) e^{-\gamma(s-t)} ds, \tag{41}$$

we can rewrite (40) as the following dynamic system,

$$\dot{\lambda}_1(t) = \delta q(t), \tag{42}$$

$$\dot{q}(t) = \gamma q(t) - (\ln(w_1(t)) - \ln(w_2(t))) = \gamma q(t) - \ln\left(\frac{w_1(t)}{w_2(t))}\right). \tag{43}$$

where $\lambda_1(0)$ is known and $\lim_{t\to\infty} q(t) = 0$. Notice that $\dot{\lambda}_2(t) = -\dot{\lambda}_1(t)$. We can rewrite (43) as:

$$\dot{q}(t) = \gamma q(t) - (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \ln\left[\frac{A_1^{\frac{1}{1-\alpha}}\left(N_1 + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_2\right)}{A_2^{\frac{1}{1-\alpha}}\left(N_2 + \Upsilon^{\frac{-\alpha}{1-\alpha}} N_1\right)}\right]. \tag{44}$$

5.3 Steady State and dynamics

We assume that once the economy reqches a steady state, N_1 and N_2 grow at constant rates g_1 and g_2 respectively. Under this assumption:

$$g_1 = \frac{\dot{N}_1}{N_1} = \frac{1 + \Upsilon^{\frac{-\alpha}{1-\alpha}}\frac{1}{z}}{\eta_1}\lambda_1$$

and

$$g_2 = \frac{\dot{N}_2}{N_2} = \frac{1 + \Upsilon^{\frac{-\alpha}{1-\alpha}} z}{\eta_2}\lambda_2,$$

where z is defined as $z = \frac{N_1}{N_2}$.

[4] FT proceed likewise in the last step of their analysis.

If λ reaches a steady state, we can conclude that if g_1 and g_2 are constant in the long-run and so does z. This implies that $g_1 = g_2 = g$ and

$$\frac{1 + \Upsilon^{\frac{-\alpha}{1-\alpha}}\frac{1}{z}}{\eta_1}\lambda_1 = \frac{1 + \Upsilon^{\frac{-\alpha}{1-\alpha}}z}{\eta_2}\lambda_2. \tag{45}$$

We can rewrite (44) as

$$\dot{q}(t) = \gamma q(t) - (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \ln\left(\frac{A_1^{\frac{1}{1-\alpha}}\left(z + \Upsilon^{\frac{-\alpha}{1-\alpha}}\right)}{A_2^{\frac{1}{1-\alpha}}\left(1 + \Upsilon^{\frac{-\alpha}{1-\alpha}}z\right)}\right). \tag{46}$$

The economy dynamics are described in the following proposition:

Proposition 3 [The Economy Dynamics]
Given the initial conditions $\{\lambda_1(0), N_1(0), N_2(0)\}$ *and a terminal condition for* q, $\lim_{t\to\infty} q(t) = 0$, *the economy dynamics are described by the following three dimensional system:*

$$\begin{cases} \dot{\lambda}_1(t) = \delta q(t), \\ \dot{q}(t) = \gamma q(t) - (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \ln\left(\frac{A_1^{\frac{1}{1-\alpha}}\left(z+\Upsilon^{\frac{-\alpha}{1-\alpha}}\right)}{A_2^{\frac{1}{1-\alpha}}\left(1+\Upsilon^{\frac{-\alpha}{1-\alpha}}z\right)}\right), \\ \dot{z} = \lambda\frac{z+\Upsilon^{\frac{-\alpha}{1-\alpha}}}{\eta_1} - (1-\lambda)\frac{z+\Upsilon^{\frac{-\alpha}{1-\alpha}}z^2}{\eta_2}. \end{cases} \tag{47}$$

If $\min\{\frac{A_1}{A_2}\} \geq \Upsilon^{-\alpha}$, *the economy possesses a unique interior steady state:*

$$\begin{cases} \bar{\lambda} = \frac{\frac{z+\Upsilon^{\frac{-\alpha}{1-\alpha}}z^2}{\eta_2}}{\frac{z+\Upsilon^{\frac{-\alpha}{1-\alpha}}z^2}{\eta_2} + \frac{z+\Upsilon^{\frac{-\alpha}{1-\alpha}}}{\eta_1}}, \\ \bar{q} = 0, \\ \bar{z} = \frac{A_2^{\frac{1}{1-\alpha}} - A_1^{\frac{1}{1-\alpha}}\Upsilon^{\frac{-\alpha}{1-\alpha}}}{A_1^{\frac{1}{1-\alpha}} - A_2^{\frac{1}{1-\alpha}}\Upsilon^{\frac{-\alpha}{1-\alpha}}}. \end{cases} \tag{48}$$

Proof: Substituting $\dot{\lambda}_1(t) = 0$, $\dot{q}(t) = 0$ and $q(t) = 0$ into (47), we obtain the values for the steady state.

Although we have obtained a dynamical system that describes the evolution of the model and its possible long-term configuration, we cannot analyse its stability properties. Besides, to ensure feasibility, we need $\min\{\frac{A_1}{A_2}\} \geq \Upsilon^{-\alpha}$ to have $\bar{z} \geq 0$. Otherwise, the steady state would not be feasible. In other words, we need our regions to be close technologically speaking in order to obtain a steady state, otherwise we cannot obtain any long-term distribution.

Following Picard and Toulemonde (2003), we provide the definition of a stable equilibrium:

Definition 3 **[Stable equilibrium]**
An equilibrium $\{\bar{\lambda}, \bar{N}_1, \bar{N}_2\}$ *is stable if, in the neighborhood of* $\bar{\lambda}$ *no locational deviation by a group of individuals is profitable.*

Applying the above definition of stability one can analyze the stability of two specific solutions, the corner solutions where all skilled individuals rest in one of the regions:

Proposition 4 **[Stability of the corner solutions]**
The two corner solutions defined by $\bar{\lambda}_1 = 0$ *et* $\bar{\lambda}_1 = 1$ *are stable solutions of (47).*

Proof: See the appendix.

Without making further assumptions, proposition 4 is the unique analitical result we can obtain, that is, we cannot study the interior steady state. In the next subsections we make some simplifications regarding regional technologies in order to further analyse the stability of the interior steady state.

5.3.1 Both regions use the same technology: $A_1 = A_2$

If both regions used the same technology in the final good production, then the number of patents that the two regions can develop is equal. This does not necessarily imply that the number of researchers must also be equal, since fixed costs could reduce one region's productivity. Therefore, if fixed costs are larger in one region than in another, more researchers would be needed to produce the same quantity of patents.

Proposition 5 **[Stability analysis]**
Under the assumption $A_1 = A_2$*, the dynamic system (47) has a unique (saddle path) steady state described by* $\bar{\lambda} = \frac{\eta_1}{\eta_1+\eta_2}$*,* $\bar{q} = 0$ *and* $\bar{z} = 1$.

Proof: See appendix 9.

In contrast with the two previous models, the present model has, technically speaking, a classical structure with a unique steady state, one forward and two backward variables which allows for a saddle-path. The multiple long-run solutions obtained in section 4 have been transformed into a unique steady state uniquely determined by initial endowments. The most remarkable difference between FT and this model lies in the production of new patents in the R&D sector. Notice that the number of researchers is the externality itself in FT whereas for Romer, researchers is an input for ideas production but it is not the productive externality. In this case, the externality comes from the accumulation of ideas, which is a backward variable of the model. Hence, the externality within the Romer economic structure is not strong enough to cause multiple equilibria.

We have ran numerical simulations to illustrate the stable arm of (47) applying the algorithm developed by Brunner and Strulik (2002). This algorithm is based on backward integration, which transforms an unstable forward looking system into a stable backward looking system and exploits this characteristic. Besides, the approximation of the infinite time horizon is endogenously determined (see appendix 11 for details).

Figure 1 shows that along the direction of the eigenvector associated to the negative eigenvalue ($v = (0, 0, 1)$) the system converges to the steady state.

α	1/3
δ	0.4
Υ	1.05
γ	0.01
η_1	0.97
η_2	0.95

Table 1: Parameter values for the numerical exercise when $A_1 = A_2$

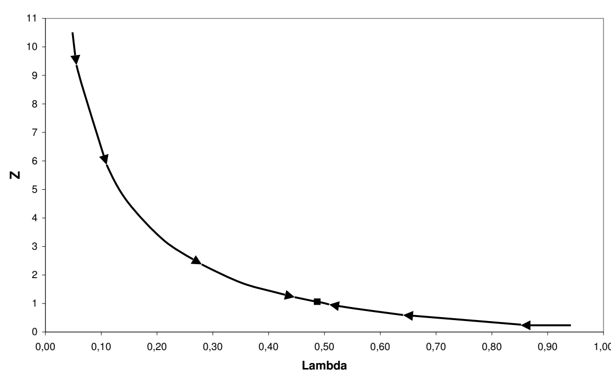

Figure 1: Stable manifold, $A_1 = A_2$

5.3.2 Regions use different technologies: $A_1 \neq A_2$

When $A_1 \neq A_2$, the characteristic polynomial is of third degree with real non-integer coefficients. Therefore, we use a numerical resolution to obtain the roots of the characteristic polynomial to infer the dynamic properties of the model. The baseline scenario coincides with that presented on Table 1, with $A_1 = 10$ and $A_2 = 9.999$.

Under these parameter values, the economy diverges from the its steady state since the three roots of the characteristic polynomial are $l_1 = -0.935$, $l_{2,3} = 0.016 \pm 0.04i$. We have considered several alternatives scenarios where we have modified the values of all parameters over a feasible range. The long-run characteristics of the system seem sensible only to parameter Υ. In effect, when $\Upsilon \to 1$ we obtain a positive, a negative and a zero eigenvalue. In this case, there exists a set of initial conditions that ensure convergence to the steady state.

We would like to illustrate the model's dynamical behavior by means of numerical simulations. We set the same parameter values as in Table 1 but we set $\Upsilon = 1.005$ to ensure the existence of a convergent path, that is of a positive, a negative and a zero eigenvalue. The steady state is $\{\bar{\lambda} = 0.48; \quad \bar{q} = 0; \quad \bar{z} = 1\}$. We choose $\lambda(0) = 0.1\bar{\lambda}$ to compute the increasing arm of the manifold and $\lambda(0) = 2\bar{\lambda}$ for the decreasing arm. The stable manifold is depicted in figure 2.

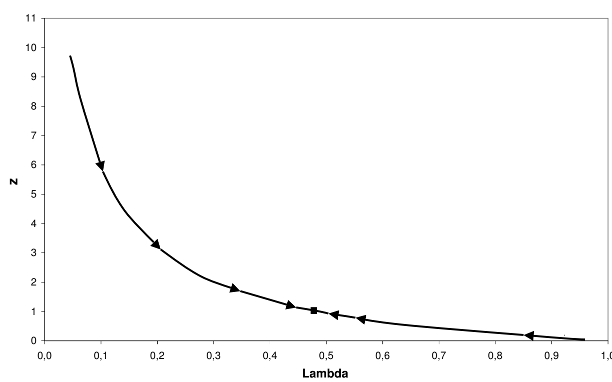

Figure 2: Stable manifold, $A_1 \neq A_2$

We have studied the behavior of the system when Υ and η_1 receive shocks. If the shock is temporary, the manifold does not lose its stability and the trajectory is modified only temporary. If the shock is permanent and agents anticipate it, the solution path shifts though it converges to the new interior steady state.

6 Conclusion

We have presented three frameworks that model inter-regional migrations. The first model, introduced by Fujita and Thisse (2002) is very complex. As a result a standard dynamic analysis is impossible. Since we find this framework potentially rich, we simplify the migration law and remove the mc-hypothesis. Having introduced these changes, we analyze the model's dynamics that are in the same line as FT's results.

Section 5 proposes a version of Romer (1990) applied to an economy with two regions and where workers are allowed to migrate. The steady state is unique and it depends on several regional parameters as fixed costs, technology but also on transportation costs. We have proved that there exists a set of initial conditions along the direction of the stable manifold for which the solution converges to the steady state. Moreover, the dynamic behavior of the model is determined by history.

While FT model is more complex and throughout, it introduces several analytical problems, which cannot be solved by merely changing the migration law. The model presented in section 5 proposes a standard multi-sectorial approach to model inter-regional migrations and it studies its consequences on economic growth.

7 Appendix: Standard approach to solve FT's dynamics

Let us rewrite the equation of motion for λ:

$$\dot{\lambda}(t) = \frac{\delta}{\gamma} e^{\gamma t} \ln\left(\frac{a_H + W(0;t)}{a_H + W(0;T)}\right) + \delta\mu e^{\gamma t} \int_t^T e^{-\gamma s} \ln\left(\frac{P_2(s)}{P_1(s)}\right) ds.$$

Let us call $\Theta(t)$ the following function:

$$\Theta(t) = e^{\gamma t} \int_t^T \ln\left(\frac{P_2(s)}{P_1(s)}\right) e^{-\gamma s} ds,$$

taking its first derivative:

$$\dot{\Theta}(t) = \gamma\Theta(t) + \ln\left(\frac{P_1(t)}{P_2(t)}\right) = \gamma\Theta(t) + \ln\left(\frac{E_1(t)}{E_2(t)}\right).$$

We shall take the derivatives of regional expenditures from

$$E_1(t) = \tfrac{L}{2} + \int_0^t \gamma\dot{\lambda}_1(s)\left(a_H + W(0,s)\right) I_{\dot{\lambda}_1(s)>0} ds + \int_t^T \gamma(-\dot{\lambda}_1(s))\left(a_H + W(0,s)\right) I_{\dot{\lambda}_1(s)<0} ds +$$

$$+\lambda_1(0)\gamma\left(a_H + W(0)\right),$$

and

$$E_2(t) = \tfrac{L}{2} + \int_0^t \gamma\dot{\lambda}_2(s)\left(a_H + W(0,s)\right) I_{\dot{\lambda}_2(s)>0} ds + \int_t^T \gamma(-\dot{\lambda}_2(s))\left(a_H + W(0,s)\right) I_{\dot{\lambda}_2(s)<0} ds +$$

$$+\lambda_2(0)\gamma\left(a_H + W(T)\right),$$

we have that

$$\dot{E}_1(t) = \gamma\dot{\lambda}_1(t)\left(a_H + W(0,t)\right) I_{\dot{\lambda}_1(t)>0} ds + \gamma\dot{\lambda}_1(t)\left(a_H + W(0,s)\right) I_{\dot{\lambda}_1(s)<0} =$$

$$= \dot{\lambda}(t)\left(a_H + \frac{W(0,t)}{\gamma}\right).$$

Similarly

$$\dot{E}_2(t) = -\dot{\lambda}(t)\left(a_H + \frac{W(0,t)}{\gamma}\right),$$

which was expected since $E_1 + E_2 = E$ is constant. Notice that we still have an integral term in the derivatives of regional expenditure: $a_H + W(0,t)$. Let us recall that

$$a_H + W(0,t) = a_H + \int_0^t w_2(s) e^{-\gamma s} ds + \int_t^\infty w_1(s) e^{-\gamma s} ds,$$

where

$$w_2(t) = k_2(\lambda(t)) \frac{\mu E}{\sigma} \int_t^\infty e^{-\int_t^\tau (\gamma + g(\lambda(s))) ds} d\tau,$$

and

$$w_1(t) = k_1(\lambda(t))\frac{\mu E}{\sigma}\int_t^{\infty} e^{-\int_t^{\tau}(\gamma+g(\lambda(s)))ds} d\tau.$$

Then, if we call $\Lambda(t) = a_H + W(0,t)$, we have that

$$\dot{\Lambda}(t) = e^{-\gamma t}\frac{\mu E}{\sigma}\left(k_2(\lambda) - k_1(\lambda)\right)\int_t^{\infty} e^{-\int_t^{\tau}(\gamma+g(\lambda(s)))ds} d\tau,$$

which also has an integral term. Let us have a more detailed look into the last integral term: $\Omega(t) = \int_t^{\infty} e^{-\int_t^{\tau}(\gamma+g(\lambda(s)))ds} d\tau$. The first questions are whether it is growing infinitely or whether on the contrary, it reaches a steady state. To study these questions, let us compute its first derivative:

$$\dot{\Omega}(t) = \left((\gamma + g(\lambda(t))\right)\Omega(t) - 1.$$

If $\lambda(t)$ reaches a steady state, then $\Omega(t) = \frac{1}{\gamma+g(\bar{\lambda})}$, which is constant. Nevertheless, we are not yet sure about the long-run behavior of Λ.

Then, we can build a dynamic system made of the following equations:

$$\begin{cases} \dot{\lambda}(t) = \frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{\Lambda(t)}{\Lambda(T)}\right) + \delta\mu\Theta(t), \\ \dot{\Theta}(t) = \gamma\Theta(t) + \ln\left(\frac{E_1(t)}{E_2(t)}\right), \\ \dot{E}_1(t) = \Lambda(t)\left(\frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{\Lambda(t)}{\Lambda(T)}\right) + \delta\mu\Theta(t)\right), \\ \dot{E}_2(t) = -\Lambda(t)\left(\frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{\Lambda(t)}{\Lambda(T)}\right) + \delta\mu\Theta(t)\right), \\ \dot{\Lambda}(t) = e^{-\gamma t}\frac{\mu E}{\sigma}\left(k_2(\lambda) - k_1(\lambda)\right)\Omega(t), \\ \dot{\Omega}(t) = \left((\gamma + g(\lambda(t))\right)\Omega(t) - 1. \end{cases} \tag{49}$$

or using that $E = E_1 + E_2$:

$$\begin{cases} \dot{\lambda}(t) = \frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{\Lambda(t)}{\Lambda(T)}\right) + \delta\mu\Theta(t), \\ \dot{\Theta}(t) = \gamma\Theta(t) - \ln\left(\frac{E}{E_1(t)} - 1\right), \\ \dot{E}_1(t) = \Lambda(t)\left(\frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{\Lambda(t)}{\Lambda(T)}\right) + \delta\mu\Theta(t)\right), \\ \dot{\Lambda}(t) = e^{-\gamma t}\frac{\mu E}{\sigma}\left(k_2(\lambda) - k_1(\lambda)\right)\Omega(t), \\ \dot{\Omega}(t) = \left((\gamma + g(\lambda(t))\right)\Omega(t) - 1. \end{cases} \tag{50}$$

We know the initial conditions for λ, E_1 and E_2: $\{\lambda(0), E_1(0), E_2(0)\}$. It is more difficult to decide which kind of condition we have for Λ. Notice that at time $t = 0$, the future values of this variable are unknown, which implies that both $\Lambda(0)$ and $\Lambda(T)$ are unknown. We cannot give an initial nor a final condition for Λ. Hence, the system is not well defined and we cannot run any numerical exercise.

Besides notice that if the economy reached a steady state at $t = T$, then we could write $\dot{\Lambda}$ after T as

$$\dot{\Lambda}(t) = e^{-\gamma t}\frac{\mu E}{\sigma}\frac{1}{\gamma + g(\overline{\lambda})}\left(k_2(\overline{\lambda}) - k_1(\overline{\lambda})\right).$$

Which is not constant. It is true that $lim_{t\to\infty}\dot{\Lambda}(t) = 0$, except if $\lambda(t) = 1/2$, $\forall t \geq 0$.

As a conclusion from this discussion, if $\lambda(t) = 1/2$ $\forall t$, then the system would always be at steady state. If not, all variables but Λ could reach a steady state value in finite time, but Λ would continue growing (if $k_2(\overline{\lambda}) - k_1(\overline{\lambda}) > 0$, decreasing otherwise). Furthermore, we cannot compute an initial nor a final condition for Λ and Λ's growth of rate is not constant, which impedes a standard dynamic analysis.

Next, we need to verify the nature of the steady state if the relationship between E_1/E_2 and ϕ, $1/\phi$ was different, that is if we were at another equilibrium.

If $E_1/E_2 \geq 1/\phi$, then $M_1 = M$ and $M_2 = 0$. Then, using (8), we have that $P_2/P_1 = (1/\phi)^{\frac{1}{\sigma-1}}$. If we replace the price ratio into the migration law, we obtain that:

$$\dot{\lambda}(t) = \frac{\delta}{\gamma}e^{\gamma t}\ln\left(\frac{a_H + W(0;t)}{a_H + W(0;T)}\right) + \frac{\delta\mu}{\gamma}\ln(1/\phi)^{\frac{1}{\sigma-1}} - \frac{\delta\mu}{\gamma}\ln(1/\phi)^{\frac{1}{\sigma-1}}e^{\gamma(t-T)}.$$

As in the equilibrium where $1/\phi < E_1/E_2 < \phi$, $\dot{\lambda}(T) = 0$ by construction. However, what is new now is that this problem does not have $\lambda = 1/2$ as a steady state. It may have other values but they are not trivial to find. Then, we conclude as in the previous analysis, that the long-run value for λ is not determined. We face the same problems as before when trying to analyze the steady state properties or long-run properties of the dynamical system.

8 Appendix: Computation of the Jacobian of the dynamic system obtained in section 4.1

We compute the Jacobian of (23):

$$J(\lambda, q, V, t) = \begin{pmatrix} 0 & \beta & 0 \\ -\gamma\left(k_1'(\lambda) - k_2'(\lambda)\right) & \gamma & k_1(\lambda) - k_2(\lambda) \\ g'(\lambda)V & 0 & \gamma + g(\lambda) \end{pmatrix}$$

If we substitute for the steady state values:

$$J(\overline{\lambda}, \overline{q}, \overline{V}) = \begin{pmatrix} 0 & \beta & 0 \\ -2\gamma(1-\eta)k'(\frac{1+\eta}{2}) & \gamma & 0 \\ g'(1/2)V & 0 & \gamma + g(1/2) \end{pmatrix}$$

Computing $|J(\overline{\lambda}, \overline{q}, \overline{V}) - lI| = 0$, we obtain the values of the three eigenvalues of the system: $l_1 = \gamma + g(1/2)$ and $l_{2,3} = \frac{\gamma \pm \sqrt{\gamma^2 - 8\beta(1-\eta)\gamma k'(\frac{1+\eta}{2})}}{2}$.

9 Appendix: Inter-regional migration within a Romer structure. The Jacobian of the dynamic system.

Under the assumption $A_1 = A_2$, the Jacobian of (47) is

$$J(\lambda, q, z, t) = \begin{pmatrix} 0 & \beta & 0 \\ 0 & \gamma & 0 \\ \frac{1+\Upsilon^{\frac{-\alpha}{1-\alpha}}}{\eta_1} + \frac{1+\Upsilon^{\frac{-\alpha}{1-\alpha}}}{\eta_2} & 0 & \frac{\lambda}{\eta_1} - (1-\lambda)\frac{1+2\Upsilon^{\frac{-\alpha}{1-\alpha}}}{\eta_2} \end{pmatrix}$$

If we develop $|J - l \quad I| = 0$, we obtain that the system eigenvalues are: $l_1 = 0$, $l_2 = \gamma$, $l_3 = \frac{\overline{\lambda}}{\eta_1} - (1-\lambda)\frac{1+2\theta_2}{\eta_2}$. To obtain the values for the eigenvalues exposed in the proposition, we only need to substitute $\overline{\lambda}$ by its value in equation (48).

10 Appendix: Stability of the corner solutions

If $\bar{\lambda} < \lambda(t) < 1$, then $\bar{\lambda} = 1$ would be stable if and only if $\dot{\lambda}(t) > 0$. $\dot{\lambda}(t) > 0$ if and only if $q > 0$. Using the definition of q in (41), a necessary condition for $q > 0$ is that $w_1 > w_2$. $w_1 > w_2$ if and only if $z > \bar{z}$. Hence, $\dot{\lambda}(t) > 0$ if and only if $z > \bar{z}$. Using the definition of $\dot{z}$, we obtain that $\dot{z} > 0$ if and only if $\bar{\lambda} < \lambda(t)$.

Summarizing, if $\bar{\lambda} < \lambda(t) < 1$ then $\dot{z} > 0$ and $z > \bar{z}$. This in turn implies that $\dot{labmda} > 0$ which proves that $\bar{\lambda} = 1$ is stable. It can be proved, following the same reasoning as above, that $\bar{\lambda} = 0$ is also a stable corner solution.

11 Appendix: Backward integration algorithm by Brunner and Strulik (2002).

Let us assume that the original system is

$$\frac{d}{dt}(x(t), y(t))^T = F(x(t), y(t)) \tag{51}$$

with $x \in \mathbb{R}^l$, $y \in \mathbb{R}^m$ and the initial value of y, $y(0)$ is known. Besides, (51) with the initial condition for y satisfies a boundary condition:

$$\lim_{t\to\infty}(x(t), y(t))^T = (x^*, y^*)^T,$$

where $(x^*, y^*)^T$ is the steady state of (51). Furthermore, the system made of (51) together with the initial condition for y and the boundary condition has a unique solution. Therefore, the value of $x(0)$ is unknown though determined by the other elements of the system.

The idea of backward integration is to obtain the solution of a dynamical system starting from a value close to the steady state towards the initial condition. That is, time is reversed by the following change of variable $\tilde{t} = -t$. If the dynamics of the original model were defined by (51), then after the change of variable, they become

$$\frac{d}{dt}(x(-\tilde{t}), y(-\tilde{t}))^T = -F(x(-\tilde{t}), y(-\tilde{t})). \tag{52}$$

Let us denote $\tilde{x} =: x(-\tilde{t})$. Multiplying by -1 converts the stable manifold of (51) into an unstable manifold of (52) and viceversa. The unstable manifold

$$V_u = \{(x(t), y(t))^T / \lim_{t\to-\infty}(x(t), y(t))^T = (x^*, y^*)^T\}$$

of (52) is the stable manifold of (51). Since V_u contains the convergent solution to (51), we can choose an initial value

$$(x(0), y(0))^T \in V_u \tag{53}$$

close to the steady state $(x^*, y^*)^T$. This forces the solution of (52) to take the same trajectory as the solution of (51) but in reversed direction, and it takes the value y_0 for $t_N > 0$.

One important feature of the algorithm is that it uses the fact that trajectories are equivalent if the solution to (51) is truncated at time t_N, which is endogenously determined.

Starting from (53) we need to choose a direction of departure from the steady state. This method proposed to compute the Jacobian of (51) evaluated at the steady state and the associated set of eigenvalues. Since y is of dimension l, we must obtain at least l negative eigenvalues. The associated eigenvectors generate a tangential subspace to V_u. If $l = 1$, then the starting point would be

$$(\tilde{x}, \tilde{y})^T = (x^*, y^*)^T + \epsilon v_1,$$

where v_1 is the eigenvector associated to the negative eigenvalue. If $l = 2$, they propose:

$$(\tilde{x}, \tilde{y})^T = (x^*, y^*)^T + \epsilon v_1 \sin\theta + \epsilon v_2 \cos\theta,$$

with $\theta \in [0, 2\pi)$ and $\epsilon \in \mathbb{R}$.

To summarize the algorithm, we can describe it in three steps:

Step 1: Enter $-F$ and the starting value.

Step 2: Enter $y(0)$ as a stopping criterion. Then for one $n \in \mathbb{N}$, $y^n \approx y(0)$. We must introduce a maximum error of discretization.

Then, we run the backward integration program. If the sequence does not approach $y(0)$, introduce a new starting value and run the backward integration program again.

The solution is a three-upla $(\tilde{x}^i, \tilde{y}^i, \tilde{t}^i)_{i=1}^n$ which is the numerical solution of (52) and n is its length.

Step 3: We need to arrange the solution ot be a forward solution to 51:

$$\begin{aligned} x^i &= \tilde{x}^{n-i+1}, \\ y^i &= \tilde{y}^{n-i+1}, \\ t^i &= \tilde{t}^n - \tilde{t}^{n-i+1}, \end{aligned}$$

for $i = 1, ..., n$.

References

Antolin P., Bover O., (1997), "Regional Migration in Spain: the effect of personal characteristics and of unemployment, wage and house price differentials using pooled cross-sections", *Oxford Bulletin of Economics and Statistics*, 59, 2.

Beine M., Docquier F. and Rapoport (2001), "Brain drain and economic growth: theory and evidence", *Journal of Development Economics*, 64, 275–289.

Brunner M. and Strulik H. (2002), "Solution of perfect foresight saddlepoint problems: a simple method and applications", *Journal of Economic Dynamics and Control*, 26, 737–53.

Fujita, M. and JF. Thisse (2002), *Economics of agglomeration.* Cambridge University Press.

Fukao K. and Benabou R. (1993), "History versus expectations: a comment", *The Quarterly Journal of Economics*, 108, 535–42.

Gianetti M. (2001), "Skill Complementarities and Migration Decisions", *Labour*, 15 (1).

Gianetti M. (2003), "On the mechanics of migration decisions: skill complementarities and endogenous price differentials", *Journal of Development Economics*, 71, 329–49.

Greenwood M.J. (1997), "Internal Migration in Developed Countries", chapter 12 in *Handbook of Population and Family Economics.*

Kremer M.(1993), "The O-ring Theory of Economic Development", *Quarterly Journal of Economics*, 108, 551–76.

Krugman, P. (1991), *Increasing returns and economic geography*, Journal of Political Economy 99, 483-499.

Ledent J., (1990),"3. Canada" p. 47-61 in C. Nam, W. Serow and D. Sly, eds. *International Handbook of Internal Migration.* Westport, Connecticut: Greenwood Press.

Matsuyama K. (1991), "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium", *Quarterly Journal of Economics*, 106, 2, 617–50.

Mossay, P. (2001), *Market Structure and migration in a spatial economy*, PhD dissertation, Universit catholique de Louvain.

Mountford A. (1997), "Can a brain drain be good for growth in the source economy?", *Journal of Development Economics*, 53, 287–303.

Mussa M., (1978), "Dynamic Adjustment in the Heckscher-Ohlin-Samuelson model", *Journal of Political Economy*, 86, no.5.

Picard P.M., Toulemonde (2003), "Regional Assymmetries: Economies of agglomeration versus unionized labor markets", *Regional Science and Urban Economies*, 33, pp. 223-249.

Romer P.M. (1990), “Endogenous Technological Change”, *Journal of Political Economy*, 98, no. 5.

Stark O. and Wang Y. (2002), “Inducing human capital formation: migration as a substitute for subsidies”, *JOurnal of Public Economics*, 86, 29–46.

Vidal J.P. (1998), “The effect of emigration on human capital formation”, *Journal of Population Economics*, 11, pp. 589-600.

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

IRES

ISSN 1379-244X D/2010/3082/004